OCR
RECOGNISING ACHIEVEMENT

Wednesday 23 January 2013 – Morning

A2 GCE MATHEMATICS

4723/01 Core Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4723/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hours 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 For each of the following curves, find the gradient at the point with x-coordinate 2.

(i) $y = \dfrac{3x}{2x+1}$ **[3]**

(ii) $y = \sqrt{4x^2 + 9}$ **[3]**

2 The acute angle A is such that $\tan A = 2$.

(i) Find the exact value of $\operatorname{cosec} A$. **[2]**

(ii) The angle B is such that $\tan(A + B) = 3$. Using an appropriate identity, find the exact value of $\tan B$. **[3]**

3 **(a)** Given that $|t| = 3$, find the possible values of $|2t - 1|$. **[3]**

(b) Solve the inequality $|x - \sqrt{2}| > |x + 3\sqrt{2}|$. **[4]**

4 The mass, m grams, of a substance is increasing exponentially so that the mass at time t hours is given by

$$m = 250e^{0.021t}.$$

(i) Find the time taken for the mass to increase to twice its initial value, and deduce the time taken for the mass to increase to 8 times its initial value. **[3]**

(ii) Find the rate at which the mass is increasing at the instant when the mass is 400 grams. **[3]**

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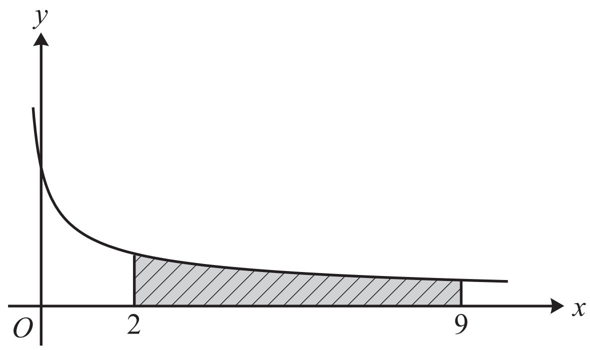


The diagram shows the curve $y = \dfrac{6}{\sqrt{3x+1}}$. The shaded region is bounded by the curve and the lines $x = 2$, $x = 9$ and $y = 0$.

(i) Show that the area of the shaded region is $4\sqrt{7}$ square units. **[4]**

(ii) The shaded region is rotated completely about the x-axis. Show that the volume of the solid produced can be written in the form $k\ln 2$, where the exact value of the constant k is to be determined. **[5]**

6 **(i)** By sketching the curves $y = \ln x$ and $y = 8 - 2x^2$ on a single diagram, show that the equation

$$\ln x = 8 - 2x^2$$

has exactly one real root. **[3]**

(ii) Explain how your diagram shows that the root is between 1 and 2. **[1]**

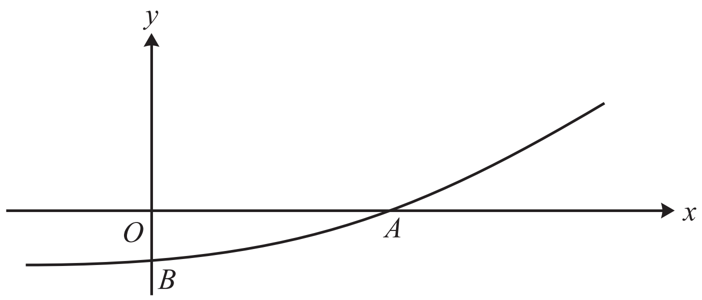
(iii) Use the iterative formula

$$x_{n+1} = \sqrt{4 - \tfrac{1}{2}\ln x_n},$$

with a suitable starting value, to find the root. Show all your working and give the root correct to 3 decimal places. **[4]**

(iv) The curves $y = \ln x$ and $y = 8 - 2x^2$ are each translated by 2 units in the positive x-direction and then stretched by scale factor 4 in the y-direction. Find the coordinates of the point where the new curves intersect, giving each coordinate correct to 2 decimal places. **[3]**

7



The diagram shows the curve with equation

$$x = (y + 4)\ln(2y + 3).$$

The curve crosses the x-axis at A and the y-axis at B.

(i) Find an expression for $\frac{dx}{dy}$ in terms of y. **[3]**

(ii) Find the gradient of the curve at each of the points A and B, giving each answer correct to 2 decimal places. **[5]**

8 The functions f and g are defined for all real values of x by

$$f(x) = x^2 + 4ax + a^2 \quad \text{and} \quad g(x) = 4x - 2a,$$

where a is a positive constant.

(i) Find the range of f in terms of a. **[4]**

(ii) Given that $fg(3) = 69$, find the value of a and hence find the value of x such that $g^{-1}(x) = x$. **[6]**

9 **(i)** Prove that

$$\cos^2(\theta + 45°) - \tfrac{1}{2}(\cos 2\theta - \sin 2\theta) \equiv \sin^2\theta.$$ **[4]**

(ii) Hence solve the equation

$$6\cos^2(\tfrac{1}{2}\theta + 45°) - 3(\cos\theta - \sin\theta) = 2$$

for $-90° < \theta < 90°$. **[3]**

(iii) It is given that there are two values of θ, where $-90° < \theta < 90°$, satisfying the equation

$$6\cos^2(\tfrac{1}{3}\theta + 45°) - 3(\cos\tfrac{2}{3}\theta - \sin\tfrac{2}{3}\theta) = k,$$

where k is a constant. Find the set of possible values of k. **[3]**

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Question			Answer	Marks	Guidance
1	(i)		Either Attempt use of quotient rule	M1	allow numerator wrong way round but needs minus sign in numerator and both terms in numerator involving x; for M1 condone minor errors such as absence of square in denominator, absence of brackets, …
			Obtain $\frac{3(2x+1)-6x}{(2x+1)^2}$ or equiv	A1	give A0 if necessary brackets absent unless subsequent calculation indicates their 'presence'
			Substitute 2 to obtain $\frac{3}{25}$ or 0.12	A1	or simplified equiv but A0 for final $\frac{3}{5^2}$
				[3]	
			Or Attempt use of product rule for $3x(2x+1)^{-1}$	M1	allow sign error; condone no use of chain rule
			Obtain $3(2x+1)^{-1}-6x(2x+1)^{-2}$ or equiv	A1	
			Substitute 2 to obtain $\frac{3}{25}$ or 0.12	A1	or simplified equiv
1	(ii)		Differentiate to obtain form $kx(4x^2+9)^n$	M1	any non-zero constants k and n (including 1 or $\frac{1}{2}$ for n)
			Obtain $4x(4x^2+9)^{-\frac{1}{2}}$	A1	or (unsimplified) equiv
			Substitute 2 to obtain $\frac{8}{5}$ or 1.6	A1	or simplified equiv but A0 for final $\frac{8}{\sqrt{25}}$
				[3]	
2	(i)		Either Attempt to find exact value of $\sin A$	M1	using right-angled triangle or identity or …
			Obtain $\frac{1}{2}\sqrt{5}$ or $\sqrt{\frac{5}{4}}$ or exact equiv	A1	final $\pm\frac{1}{2}\sqrt{5}$ is A0; correct answer only earns M1A1
				[2]	
			Or Attempt use of identity $1+\cot^2 A = \operatorname{cosec}^2 A$	M1	using $\cot A = \frac{1}{2}$; allow sign error in attempt at identity
			Obtain $\frac{1}{2}\sqrt{5}$ or $\sqrt{\frac{5}{4}}$ or exact equiv	A1	final $\pm\frac{1}{2}\sqrt{5}$ is A0; correct answer only earns M1A1
2	(ii)		State or imply $\frac{2+\tan B}{1-2\tan B}=3$	B1	
			Attempt solution of equation of form $\frac{\text{linear in } t}{\text{linear in } t}=3$	M1	by sound process at least as far as $k\tan B = c$
			Obtain $\tan B = \frac{1}{7}$	A1	answer must be exact; ignore subsequent attempt to find angle B
				[3]	

Question			Answer	Marks	Guidance
3	(a)		Substitute $t=3$ in $\|2t-1\|$ and obtain value 5	B1	not awarded for final $\|5\|$ nor for ± 5
			Substitute $t=-3$ in $\|2t-1\|$ and apply modulus correctly to any negative value to obtain a positive value	M1	with no modulus signs remaining
			Obtain value 7 as final answer	A1	not awarded for final $\|7\|$ nor for ± 7 NB: substitutions in $\|2t+1\|$ will give 5 and 7 – this is 0/3, not MR; a further step to $5<t<7$ – B1 M1 A0; answers $\pm 5, \pm 7$ – this is B0 M0 A0
				[3]	
3	(b)		<u>Either</u> Attempt solution of linear equation or inequality with signs of x different	M1	
			Obtain critical value $-\sqrt{2}$	A1	or equiv (exact or decimal approximation)
			<u>Or 1</u> Attempt to square both sides	M1	obtaining at least 3 terms on each side
			Obtain $x^2-2\sqrt{2}\,x+2>x^2+6\sqrt{2}\,x+18$	A1	or equiv; or equation; condone $>$ here
			<u>Or 2</u> Attempt sketches of $y=\|x-\sqrt{2}\|$, $y=\|x+3\sqrt{2}\|$	M1	
			Obtain $x=-\sqrt{2}$ at point of intersection	A1	or equiv
			Conclude with inequality of one of the following types: $x<k\sqrt{2},\quad x>k\sqrt{2},\quad x<\frac{k}{\sqrt{2}},\quad x>\frac{k}{\sqrt{2}}$	M1	any integer k
			Obtain $x<-\sqrt{2}$ or $-\sqrt{2}>x$ as final answer	A1	final answer $x<-\frac{2}{\sqrt{2}}$ (or similar unsimplified version) is A0
				[4]	

Question			Answer	Marks	Guidance
4	(i)		Attempt process involving logarithm to solve $e^{0.021t} = 2$ Obtain 33 State (or calculate separately to obtain) 99	M1 A1 B1√ **[3]**	with t the only variable; at least as far as $0.021t = \ln 2$; must be …= 2 or greater accuracy; ignore absence of, or wrong, units; final answer $\frac{\ln 2}{0.021}$ is A0 following previous answer; no need to include units
4	(ii)		Differentiate to obtain $ke^{0.021t}$ Obtain $250 \times 0.021\,e^{0.021t}$ Substitute to obtain 8.4 or $\frac{42}{5}$	M1 A1 A1 **[3]**	where k is any constant not equal to 250 or simplified equiv $5.25e^{0.021t}$ or value rounding to 8.4 with no obvious error
5	(i)		Integrate to obtain form $k(3x+1)^{\frac{1}{2}}$ Obtain $4(3x+1)^{\frac{1}{2}}$ Apply the limits and subtract the right way round Obtain $4\sqrt{28} - 4\sqrt{7}$ and show at least one intermediate step in confirming $4\sqrt{7}$	*M1 A1 M1 A1 **[4]**	any non-zero constant k or (unsimplified) equiv; or $4u^{\frac{1}{2}}$ following substitution dep *M AG; necessary detail required; decimal verification is A0; $[...]_2^9 = 4\sqrt{28} - 4\sqrt{7} = 4\sqrt{7}$ is A0; $[...]_2^9 = 8\sqrt{7} - 4\sqrt{7} = 4\sqrt{7}$ is A0
5	(ii)		State or imply volume is $\int \pi \left(\frac{6}{\sqrt{3x+1}}\right)^2 dx$ or equiv Integrate to obtain $k\ln(3x+1)$ Obtain $12\pi\ln(3x+1)$ or $12\ln(3x+1)$ Substitute limits correct way round and show each logarithm property correctly applied Obtain $24\pi\ln 2$	B1 M1 A1 M1 A1 **[5]**	merely stating $\int \pi y^2\, dx$ not enough; condone absence of dx; no need for limits yet; π may be implied by its later appearance any non-zero constant with or without π or unsimplified equiv allowing correct applications to incorrect result of integration providing natural logarithm involved; evidence of $\ln 28 - \ln 7 = \frac{\ln 28}{\ln 7}$ error means M0 no need for explicit statement of value of k

Question			Answer	Marks	Guidance
6	(i)		Sketch more or less correct $y = \ln x$	B1	existing for positive and negative y; no need to indicate (1, 0); ignore any scales given on axes; condone graph touching y-axis but B0 if it crosses y-axis
			Sketch more or less correct $y = 8 - 2x^2$	B1	(roughly) symmetrical about y-axis; extending, if minimally, into quadrants for which $y < 0$; no need to indicate (± 2, 0), (0, 8); assess each curve separately
			Indicate intersection by some mark on diagram (just a 'blob' sufficient) of by statement in words away from diagram	B1 **[3]**	needs each curve to be (more or less) correct in the first quadrant and on curves being related to each other correctly there
6	(ii)		Refer, in some way, to graphs crossing x-axis at $x = 1$ and $x = 2$ and that intersection is between these values	B1 **[1]**	AG; the values 1 and 2 may be assumed from part (i) if clearly marked there; dependent on curves being (more or less) correct in first quadrant; carrying out the sign-change routine is B0
6	(iii)		Obtain correct first iterate	B1	to at least 3 dp (except in the case of starting value 1 leading to 2)
			Show correct iterative process	M1	involving at least 3 iterates in all; may be implied by plausible converging values
			Obtain at least 3 correct iterates	A1	allowing recovery after error; iterates given to at least 3 dp; values may be rounded or truncated
			Conclude with 1.917	A1 **[4]**	answer required to exactly 3 dp; answer only with no evidence of process is 0/4
			$1 \to 2 \to 1.91139 \to 1.91731... \to 1.91690... \to 1.91693...$ $1.5 \to 1.94865... \to 1.91479... \to 1.91707... \to 1.91692...$ $2 \to 1.91139... \to 1.91731... \to 1.91690... \to 1.91693...$		
6	(iv)		Obtain 3.92 or greater accuracy	B1√	following their answer to part (iii)
			Attempt $4 \times \ln$(part (iii) answer)	M1	
			Obtain y-coordinate 2.60	A1 **[3]**	value required to exactly 2 dp (so A0 for 2.6 and 2.603)

Question			Answer	Marks	Guidance
7	(i)		Attempt use of product rule	M1	to produce expression of form (something non-zero) $\ln(2y+3)$ + $\frac{\text{linear in } y}{\text{linear in } y}$; ignore what they call their derivative
			Obtain $\ln(2y+3)$ …	A1	with brackets included
			Obtain … + $\frac{2(y+4)}{2y+3}$	A1	with brackets included as necessary
				[3]	
7	(ii)		Substitute $y=0$ into attempt from part (i) or into their attempt (however poor) at its reciprocal	M1	
			Obtain 0.27 for gradient at A	A1	or greater accuracy 0.26558…; beware of 'correct' answer coming from incorrect version $\ln(2y+3)+\frac{8}{3}$ of answer in part (i)
			Attempt to find value of y for which $x=0$	M1	allowing process leading only to $y=-4$
			Substitute $y=-1$ into attempt from part (i) or into their attempt (however poor) at its reciprocal	M1	
			Obtain 0.17 or $\frac{1}{6}$ for gradient at B	A1	or greater accuracy 0.16666…; value following from correct working
				[5]	
8	(i)		Attempt completion of square at least as far as $(x+2a)^2$ or differentiation to find stationary point at least as far as linear equation involving two terms	*M1	or equiv but a must be present
			Obtain $(x+2a)^2-3a^2$ or $(-2a,-3a^2)$	A1	
			Attempt inequality involving appropriate y-value	M1	dep *M; allow <, > or $\le$ here; allow use of x; or unsimplified equiv
			State $y \ge -3a^2$ or $f(x) \ge -3a^2$	A1	now with $\ge$; here $x \ge -3a^2$ is A0
				[4]	

Question			Answer	Marks	Guidance
8	(ii)		Attempt composition of f and g the right way round	*M1	algebraic or (part) numerical; need to see $4x-2a$ replacing x at least once
			Obtain or imply $16x^2-3a^2$ or $144-3a^2$	A1	or less simplified equiv but with at least the brackets expanded correctly
			Attempt to find a from $\text{fg}(3)=69$	M1	dep *M
			Obtain at least $a=5$	A1	
			Attempt to solve $4x-10=x$ or $\frac{1}{4}(x+10)=x$ or $4x-10=\frac{1}{4}(x+10)$	M1	for their a; must be linear equation in one variable; condone sign slip in finding inverse of g
			Obtain $\frac{10}{3}$	A1	and no other answer
				[6]	
9	(i)		State $\cos\theta\cos 45-\sin\theta\sin 45$	B1	or equiv including use of decimal approximation for $\frac{1}{\sqrt{2}}$
			Use correct identity for $\sin 2\theta$ or $\cos 2\theta$	B1	must be used; not earned for just a separate statement
			Attempt complete simplification of left-hand side	M1	with relevant identities but allowing sign errors, and showing two terms involving $\sin\theta\cos\theta$
			Obtain $\sin^2\theta$	A1	AG; necessary detail needed
				[4]	
9	(ii)		Use identity to produce equation of form $\sin\frac{1}{2}\theta=c$	M1	condoning single value of constant c here (including values outside the range -1 to 1); M0 for $\sin\theta=c$ unless value(s) are subsequently doubled
			Obtain 70.5 or 70.6	A1	or greater accuracy 70.528…
			Obtain –70.5 or –70.6	A1√	or greater accuracy –70.528…; following first answer; and no other answer between –90 and 90; answer(s) only : 0/3
				[3]	
9	(iii)		State or imply $6\sin^2\frac{1}{3}\theta=k$	B1	
			Attempt to relate k to at least $6\sin^2 30°$	M1	
			Obtain $0<k<\frac{3}{2}$	A1	condone use of ≤
				[3]	